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VIRTUAL COACHING CLASSES
ORGANISED BY BOS (ACADEMIC), ICAI

FOUNDATION LEVEL
PAPER 3: BUSINESS MATHEMATICS, LOGICAL REASONING & STATISTICS
(REVISION SESSION - 1)

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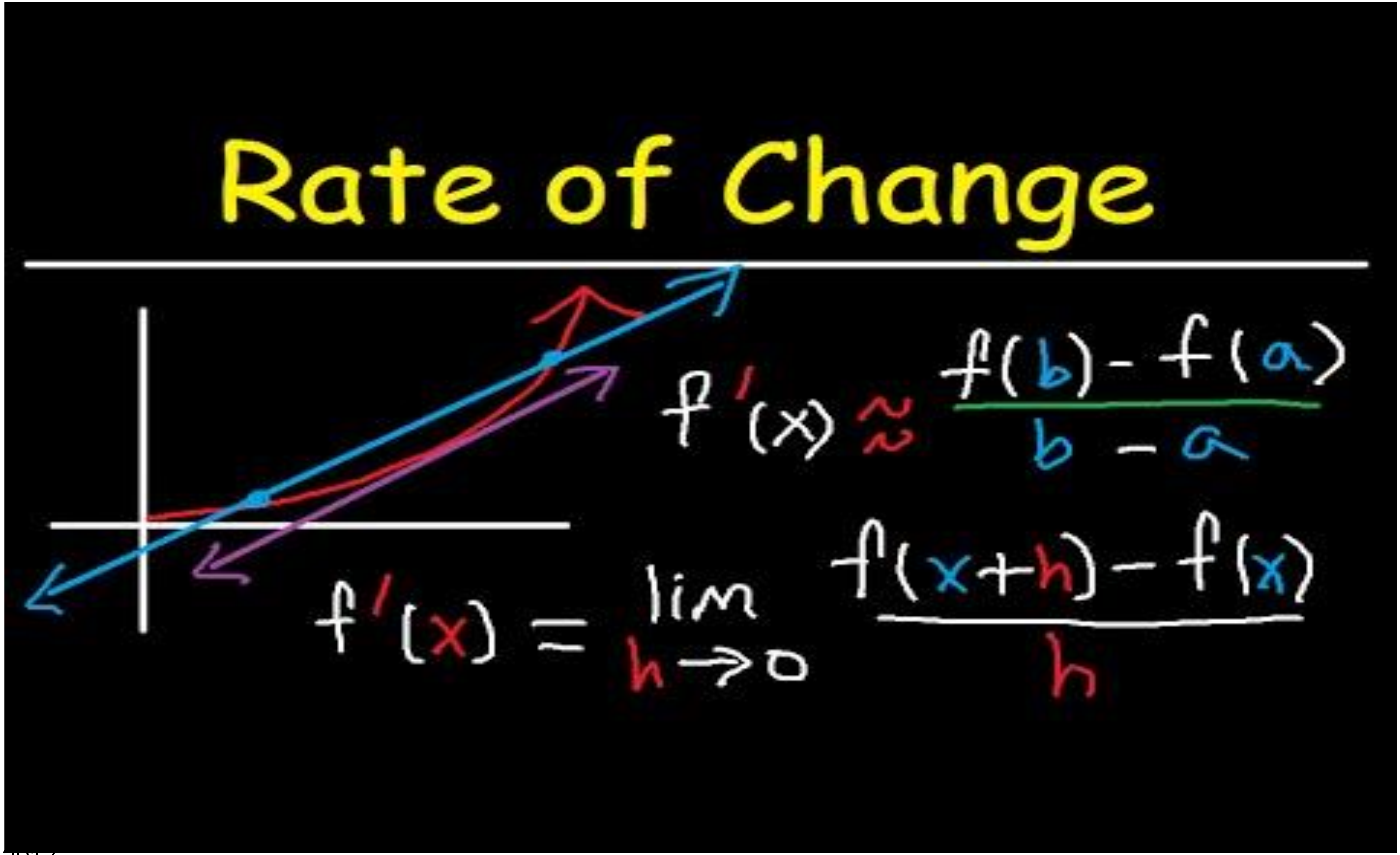


Rate of Change
$f'(x) \approx \frac{f(b)-f(a)}{b-a}$
$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$$\frac{df}{dt} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$\frac{d}{dx}(3x^2 - 2x + 1)$$

$$= 3\frac{d}{dx}(x^2) - 2\frac{d}{dx}(x) + \frac{d}{dx}(1)$$

$$= 3(2x) - 2(1) + (0)$$

$$= 6x - 2$$

Product rule

Product Rule

If $f(x)$ and $g(x)$ are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$
$$= f(x)g'(x) + g(x)f'(x)$$

or

Let $u = f(x)$ and $v = g(x)$ then

$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}$$

Pg 8.8

- **Example:** Differentiate log (1 + x2) wrt. x
- **Solution:** Let y = log (1 + x 2) = logt when t = 1 + x2
- $2x / (1+x^2)$

8.A.5 IMPLICIT FUNCTIONS

- A function in the form f(x, y) = 0. For example **x2y2 + 3xy + y = 0 where y cannot be directly defined as a function of x is called an implicit function of x.**
- In such case differentiation of both sides with respect of x and substitution of dy/dx = y1 gives the result.
- **Thereafter y1 may be obtained by solving the resulting equation.**

8.A.6 PARAMETRIC EQUATION

- When both the variables x and y are expressed in terms of a parameter (a third variable), the involved equations are called parametric equations.

- Dy/ dx = dy/ dt . Dt/ dx
- Example : If x = at^3, y = a / t3, find dy/ dx
- *dy*=*dy*´ *dt*=-3a ´1 = -1
- *dx* *dt* *dx* t^4 t^6

DERIVATIVE OF A FUNCTION OF FUNCTION

- **Example:** Differentiate log (1 + x2) wrt. x
- **Solution:** Let y = log (1 + x 2) = logt
- when t = 1 + x2
- Dy/ dt = 1/t * dt/dx= 1/ (1+x2) * 2x

8.A.7 LOGARITHMIC DIFFERENTIATION

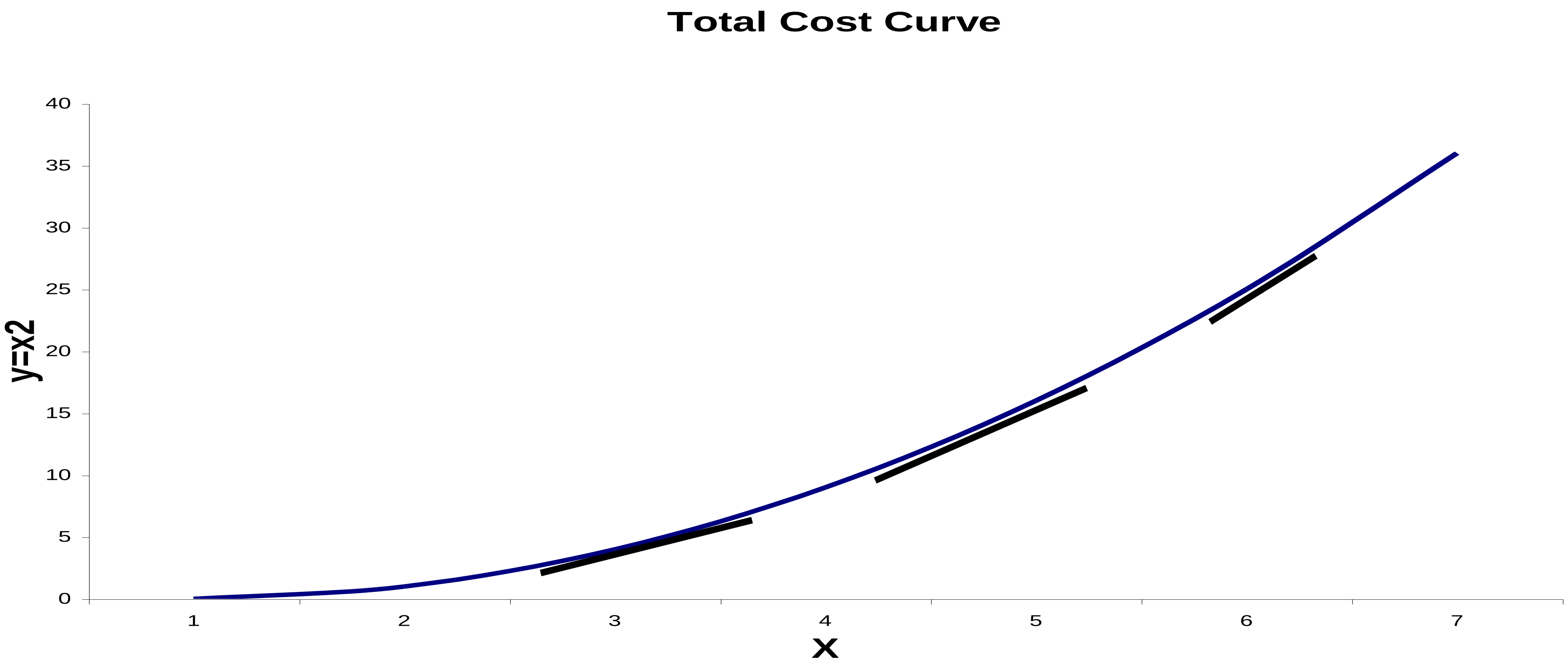
- The procedure is convenient to adopt when the function to be differentiated involves a function in its power or when the function is the product of number of functions
- **Example:** Differentiate x^x w.r.t. x
- **Solution:** let $y = x^x$ Taking logarithm, $\log y = x \log x$
- $= x^x (1 + \log x)$
- This procedure is called logarithmic differentiation.

- If $x^m y^n$ = (x+y) power m+n
- prove that dy/dx = y/x
- Taking log on both sides
- log xm yn = (m+n) log (x + y)
- or m log x + n log y = (m+n) log (x+y)
- m/x + n/y dy/dx = (m+n/ x+y) (1+ dy/dx)
- Transposing m/x to RHS and (m+n/x+y) to LHS
- Dy/ dx = y/x

8.A.9 BASIC IDEA ABOUT HIGHER ORDER DIFFERENTIATION

- Let y = f(x) = x 4 + 5x 3 + 2x2 + 9
- dy/ dx = 4x3 + 15 x2 + 4x
- d2y
- Dx2 = 12 x 2 + 30 x
- D3y/ dx3 = 24x + 30

The slope of a curve is equal to the slope of the line (or tangent) that touches the curve at that point



which is different for different values of x

- **Example:** Find the gradient of the curve y = 3x2 – 5x + 4 at the point (1, 2).
- Dy/dx = = 6x –5
- At 1,2 ----- 6.1-5= gradient is 1

Applications of Differential Calculus:

- Differentiation helps us to find out the average rate of change in the dependent variable with respect to change in the independent variable.
- It makes differentiation to have applications.
- Various scientific formulae and results involves :
- rate of change in price,
- change in demand with respect change in output,
- change in revenue obtained with respect change in price,
- change in demand with respect change in income, etc.

Pg 8.15

- **Cost Function:** Total cost consists of two parts (i) Variable Cost (ii) Fixed Cost
- If C(X) denotes the cost producing x units of a product then C(x) = V(x) + F(x) , where V(x) denotes the variable cost and F(x) is the fixed cost. Variable cost depends upon the number of units produced (i.e value of x) whereas fixed cost is independent of the level of output x. For example.
- **Average cost (AC or C) = Total cost / output**
- **Average variable cost (AVC)= V.C/ Output**
- **Average Fixed Cost (AFC) = FC/ output**

THANK YOU